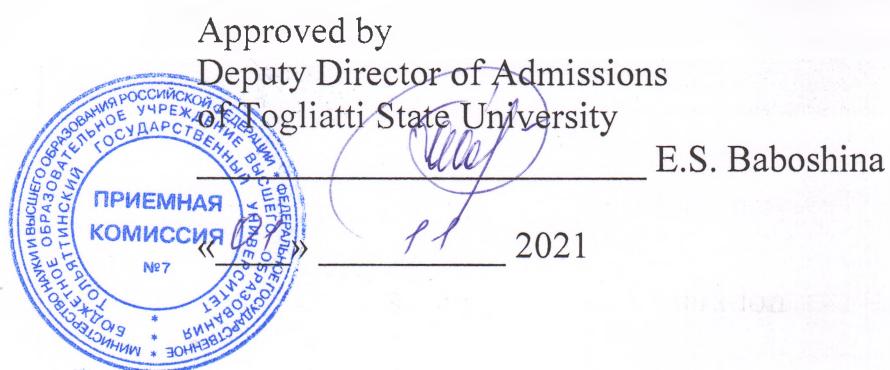Ministry of Science and Higher Education of the Russian Federation

Federal State Budgetary Educational Institution of Higher Education

Togliatti State University

Approved by
Deputy Director of Admissions
of Togliatti State University
_____ E.S. Baboshina
«____» _____ 2021

**The program of entrance examination
administered by TSU**

Mathematics

Togliatti, 2021

1. General Regulations

1.1. The entrance examination on mathematics is a computer test.

1.2. The entrance examination is for 90 minutes.

1.3. The entrance examination is scored on a 100-point scale.

2. The content of the entrance examination

2.1. Arithmetic, algebra and introduction to mathematical analysis

Arithmetic

Natural numbers (N). Prime and composite numbers. Prime factorization. Division with remainder. Power of a number with a natural exponent. Divisors and multiples of natural numbers. The greatest common divisor. The least common multiple. Divisibility rules for 2, 3, 5, 9, and 10.

Integers (Z). Coordinate line.

Rational numbers (Q). Comparison of rational numbers. Arithmetic operations with rational numbers. Properties of addition and multiplication of rational numbers. Coordinate plane. Additive inverses. Modulus of a number.

Common fractions. Equivalent fractions property. Finding fraction of a number. Finding the whole number given the value of its fraction. Proper and improper fractions. Mixed fractions. Comparison of common fractions and mixed fractions. Arithmetic operations with common fractions and mixed fractions. Decimals. Comparison and rounding of decimals. Arithmetic operations with decimals. Estimation calculation. Converting decimals to common fractions and common fractions to decimals. Infinite repeating decimals. Decimal approximation of a common fraction. Ratio. Percentage ratio of two numbers. Dividing a number in a given ratio. Proportions. The means-extremes property of proportions. Direct and inverse proportions. Percentage. Finding percentage of a number. Finding the whole number given the percentage.

Algebra

Numerical and variable expressions. The value of a numerical expression. The order of operations applied when evaluating a numerical expression. Variable expressions. Removing parentheses. Like terms. Combining like terms in an expression.

Expressions with variables. The value of an expression with variables. Permissible values of variables. Identity. Identity transformation of algebraic expressions. Verifying identities.

Power of a number with a natural exponent and its properties. Monomials. Standard form monomials. Power of a monomial. Polynomials. Standard form polynomials. Power of a polynomial. Addition, subtraction, and multiplication of polynomials. Special products of polynomials: square of a sum and square of a difference of two expressions, product of a difference and a sum of two expressions.

Factorization of polynomials. Taking the common factor out in polynomials. Grouping terms. Difference of two squares. Sum and difference of two cubes. Square trinomials. Root of a square trinomial. Properties of a square trinomial. Factorization of a square trinomial.

Rational expressions. Integer expressions. Fractional expressions. Rational fractions. Equivalent rational fractions property. Addition, subtraction, multiplication, and division of rational fractions. Raising a rational fraction to a power. Indentity transformation of rational expressions. Power of a number with an integer exponent and its properties.

Square roots. Arithmetic square root and its properties. Identity transformation of expressions containing square roots.

Equations. Equations with one variable. Root of an equation. Equivalent equations. Properties of equations with one variable. Equation as a mathematical model of a real situation. Linear equations. Quadratic equations. Quadratic formula. Vieta's formulas. Rational equations. Solving rational equations that can be transformed into linear or quadratic equations. Equations with two variables. Graph of an equation with two variables. Linear equation with two variables and its graph.

Systems of equations with two variables. Solving a system of equations with two variables by graphical method. Solving a system of equations with two variables by substitution and addition. System of equations with two variables as a model of a real situation.

Inequalities. Numerical inequalities and their properties. Addition and multiplication of numerical inequalities. Estimating the value of an expression. Inequalities with one variable. Equivalent inequalities. Number intervals. Linear and quadratic inequalities with one variable. Systems of inequalities with one variable.

Functions. Numerical functions. Functional dependencies between values. The notion of a function. Function as a mathematical model of a real process. Domain and range of a function. Methods of defining a function. Graph of a function. Plotting graphs of functions using transformation of figures. Function zeros. Constant intervals of a function. Increasing and decreasing intervals of a function.

Linear functions, inverse proportions, quadratic functions, functions of the type $y=\sqrt{x}$, their properties and graphs.

Numerical sequences. The notion of a numerical sequence. Finite and infinite sequences. Methods of defining a sequence. Arithmetic and geometric progressions. Properties of terms of arithmetic and geometric progressions. Formulas of general terms of arithmetic and geometric progressions. Formulas of sum of the first n terms of arithmetic and geometric progressions. Sum of an infinite geometric progression with $|q| < 1$. Converting infinite repeating decimals to common fractions.

Algebra and introduction to mathematical analysis
Real numbers (R) and representation of numbers on the number line. Infinite decreasing geometric progressions. Arithmetic root of a natural power. Power with a rational and real exponent.

Power function, its graph and properties. Mutually inverse functions. Equivalent equations and inequalities. Irrational equations. Irrational inequalities.

Exponential function, its graph and properties. Exponential equations. Exponential inequalities. Systems of exponential equations and inequalities.

Logarithm of a number. Cancellation property of logarithms. Logarithm of a product, a quotient, a power; change of base. Decimal and natural logarithms, e. Properties of logarithms. Logarithmic function, its graph and properties. Logarithmic equations. Logarithmic inequalities.

Trigonometric formulas. Radian measure of an angle. Rotation of a point around the origin. Definitions of a sine, cosine, and tangent of an angle. Signs of sine, cosine, and tangent. Dependency between the sine, cosine, and tangent of the same angle. Trigonometric identities. Sine, cosine, and tangent of angles α and $-\alpha$. Addition formulas. Sine, cosine, and tangent of a double angle. Sine, cosine, and tangent of a one-half angle. Trigonometric identities. Sum and difference of sines. Sum and difference of cosines.

Trigonometric equations. Equations $\sin x = a,\quad \cos x = a,\quad tgx = a$ Solving trigonometric equations. Examples of solutions to basic trigonometric inequalities.

Trigonometric functions. Domain and range of trigonometric functions. Even and odd trigonometric functions. Periods of trigonometric functions. Trigonometric functions of a numerical argument: sine, cosine, and tangent. Graphs and properties of trigonometric functions. Inverse trigonometric functions.

Definition of a derivative. Derivate of a power function. Differentiation rules. Derivatives of some basic functions. Geometric meaning of a derivative. Applying derivative to define the monotonicity of a function. Increasing and decreasing functions. Extreme values of a function. Maximum and minimum values of a function. Derivative of order two, convexity, and inflexion points. Plotting graphs.

Antiderivative. Rules for defining antiderivatives. Area of a curvilinear trapezoid. Integral. Newton-Leibniz formula.

Elements of statistics and probability. Combinatorial problems. Presenting data in tables, pie and bar charts, and graphs. Arithmetic mean. The mean value of a measure. Random event. Certain and impossible events. The probability of a random event.

2.2. Geometry. Plane geometry. Solid geometry

Plane geometry
Geometric figures. Measuring geometric magnitudes. Points. Lines. Segments. Rays. Angles. Types of angle. Complementary and vertical angles. Angle bisector. Length of a segment. Length of a polyline. Intersecting and parallel lines. Perpendicular lines. Conditions of parallelism. Properties of parallel lines. Perpendicular and oblique lines to a line.

Polygons. Triangles. Types of triangle. Median, angle bisector, altitude, midsegment of a triangle. Conditions of congruence of triangles. Properties of iso-sceles triangles. Perpendicular bisector of a line segment. The sum of triangle angles. Exterior angles of a triangle. Triangle inequality. Triangle sides and angles ratio. Pythagorean theorem. Perimeter of a polygon.

Similar triangles. Conditions of similarity of triangles. Intersection points of

medians, angle bisectors, and altitudes in a triangle. Intersection point of perpendicular bisectors of triangle sides. Properties of angle bisectors. Thales's theorem. Metric relations in a right triangle. Sine, cosine, tangent, and cotangent of an acute angle of a right triangle and angles from to 180^0. Formulas for sine, cosine, tangent, and cotangent of the same angle. Solution of triangles. Sine theorem. Cosine theorem.

Quadrangles. Parallelograms. Properties of parallelograms. Properties of rectangles, rhombuses, and squares. Trapezoids. Midsegment of a trapezoid and its properties.

Polygons. Convex polygons. Sum of angles in a convex polygon. Regular polygons.

Circles and disks. Elements of circles and disks (center, radius, diameter, chord, tangent). Center and inscribed angles. Tangent to a circle and its properties. Geometric relationship between a line and a circle. Circumscribed and inscribed circles of a triange. Circumscribed and inscribed quadrangles and their properties. Circumscribed and inscribed polygons.

Measuring geometric magnitudes. Length of a segment. Distance between two points. Distance between a point and a line. Distance between parallel lines. Perimeter of a polygon. Circumference. Circle arc length. Grade measure of an angle. Measure of an inscribed angle.

Polygon areas. Figures of equal area. Finding area of squares, rectangles, parallelograms, triangles, and trapezoids.

Disk area. Sector area. Area ratio of similar figures.

Cartesian coordinates on a plane. Formula of a distance between two points. Coordinates of the midpoint of a segment. Figure equations. Circle and line equations. Slope of a straight line.

Vectors. The notion of a vector. Modulus (magnitude) of a vector. Equal vectors. Collinear vectors. Vector coordinates. Addition and subtraction of vectors. Multiplication of a vector by a number. Scalar product of vectors. The cosine of the angle between two vectors.

Geometric transformations. Figure transformation. Figure motion. Types of figure motion: translation, axial symmetry, central inversion, rotation. Congruent figures. Homothetic dilation. Similarity of figures.

Solid geometry
Lines and planes in space. Basic terms in solid geometry (point, line, plane, and space). Intersecting, parallel, and skew lines. Angle between lines in space.

Perpendicularity of lines. Properties and conditions for parallelism and perpendicularity of a line and a plane. Theorem of three perpendiculars. Perpendicular and oblique lines to a line. Angle between a line and a plane.

Properties and conditions of parallelism and perpendicularity of planes. Dihedral angles, linear angles of a dihedral angles.

Distance between a point and a plane. Distance between a line and a plane. Distance between parallel planes. Distance between skew lines.

Polyhedrons. Vertices, edges, and faces of a polyhedron. Polyhedral angles. Convex polyhedrons.

Prism, its bases, lateral edges, altitude, and lateral surface. Right and oblique prisms. Regular prisms. Parallelepipeds. Cubes.

Pyramid, its bases, lateral edges, altitude, and lateral surface. Triangle pyramids. Regular pyramids. Truncated pyramids.

Cross section of polyhedrons. Describing cross sections.

Solids and surfaces of revolution. Cylinders and cones. Truncated cones. Base, altitude, lateral surface, generatrix, and net.

Spheres and balls, their cross sections. Ellipses, parabolas, and hyperbolas as cross sections of a cone. Plane tangent to a sphere. Sphere inscribed in a polyhedron and sphere circumscribed around a polyhedron.

Volumes and surface areas of solids. Volume ratios of similar solids. Volume formulas of cubes, parallelepipeds, prisms, and cylinders. Volume formulas of pyramids and cones. Surface area formulas of cylinders and cones. Volume formula of balls and surface area formula of spheres.

Coordinates and vectors. Cartesian coordinates in space. Formula of a distance between two points. Sphere and plane equations. Formula of distance between a point and a plane.

Vectors. Modulus of a vector. Equality of vectors. Addition of vectors and multiplication of a vector by a number. Angle between vectors. Vector coordinates. Scalar product of vectors. Collinear vectors. Resolution of a vector into two non-collinear vectors. Coplanar vectors. Resolution of a vector into three non-coplanar vectors.

Program designers

Department Head, Professor,
Doctor of Pedagogical Sciences
(position, academic title, academic degree)

R.A. Uteyeva
(Full name)

Professor, Professor,
Doctor of Pedagogical Sciences,
Candidate of Physical and Mathematical Sciences
(position, academic title, academic degree)

S.N. Dorofeyev
(Full name)

ASSESSMENT SCALE

Each of the entrance examinations administered by TSU and required for enrolment in any **Bachelor's program, Specialist's program, or Master's program** is scored on **a 100-point scale**.

$$\text{Test score in points } = \frac{\text{Examinee's raw score}}{\text{Number of test items}} \times 100,$$

where:

Test score in points is the performance of an examinee on an entrance examination (on **a 100 scale**).

The examinee's raw score is the number of correct answers an examinee gave in responding to the entrance test items.

The number of test items is the number of entrance test items for an examinee to complete.

The minimum score for successful completion of the entrance examination.